YMC 2015

M.S. Yalin Memorial Mini-Colloquium

"Fundamental river processes and connection between fluvial and coastal systems in a changing climate"

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The calculation of the **critical velocity** for the sediment motion threshold and of the **sediment discharge**: a contribution from the Russian school

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Motivation of the research

Rebuilding or expanding the industrial sector after Civil War (1918-1922) and World War II (1941-1945) in Russia led to an increase in construction and/or reconstruction of hydroelectric power plants.

Among those who took part in creating the hydrotechnical industry there were the founders of ***Fluvial Hydraulics*** in Russia: M. A. Velikanov (1849-1949), B. A Bahmetjev (1880-1952), N. N. Pavlovskii (1884-1937), A. R. Zegdza (1900-1965), V. N. Gontcarov (1900-1963), **I. I. Levi** (1900-1965), and some representative of the next generation such as **B. Studenitcnikov** (1921-1978) and A. D. Girgidov (1939).

LEVI'S FORMULA FOR SEDIMENT DISCHARGE

A number of formulae were proposed for the prediction of sediment transport. Many of them have been deduced by laboratory experiments, but they are useful also for field conditions since they incorporate dimensionless numbers.
One of these formulae, widely used in the Russian literature but not well known in the Western one, is that proposed by I. I. Levi (1948).

Prof. Ivan Ivanovich Levi was a leading representative of the Russian school of Hydraulics, mostly involved in fluvial processes. Graduated from St. Petersburg (then Leningrad) Polytechnic Institute in 1924, he spent all his work life there and in the "B. E. Vedeneev VNIIG", the leading research institution in Russia, where he created there a laboratory of fluvial processes in 1931.
In the Leningrad Polytechnic Institute from 1931 to 1951 he was vice-rector, dean of the hydrotechnical faculty, and head of the "Hydrology" department and of the laboratory of fluvial dynamics.
He got several awards for his theoretical studies, widely used for practical solutions in the construction of hydropower plants in main Russian rivers.
His research results were published only in Russian and did not obtain a recognition in the Western literature.

07.07.1900 St. Petersburg – 03.10.1975 Leningrad

Levi's formula for sediment transport is one of his major contibutions. It was derived according to the reasoning that follows.

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Background (1)

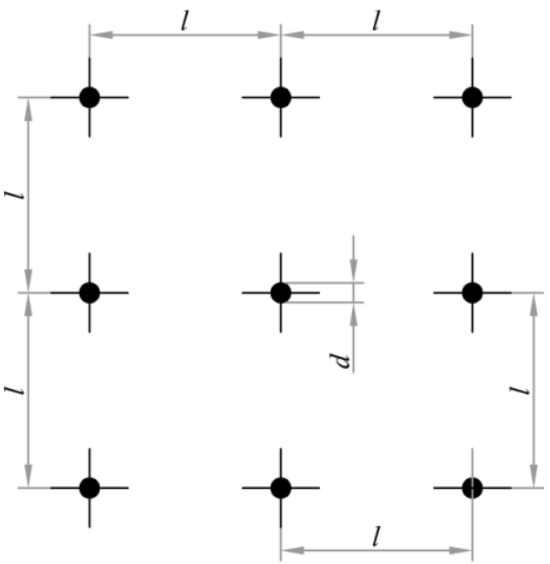
Hp: steady uniform flow
flat mobile bottom
uniform size and non-cohesive solid particles

The particles displace themselves under the action of the flow, subject to the hydrodynamic force and their submerged weight.

According to Levi, sediment discharge (in volume units) can be defined as the number of particles crossing the channel cross section in unit time, multiplied by the particle volume:

$$Q_s = b \cdot q_s = \frac{N \cdot W}{t}$$

t time
b cross-section width
q_s sediment discharge per unit width (in volume units)
N number of particles passing the cross section in t
W volume of the single particle



Sketch of the particle distribution

Background (2)

Assuming with Levi that sediment particles move with constant velocity V_S, keeping a constant distance from each other equal to l, both in the longitudinal and transversal direction, on the width b there will be $n_1 = b/l$ particles.

The number of particles on a single line which cross the transversal section in time t can be determined as the ratio between the distance they walk in time t and the distance between two next following particles: $n_2 = V_S \, t/l$.

The total number of particles crossing b is, thus, equal to:

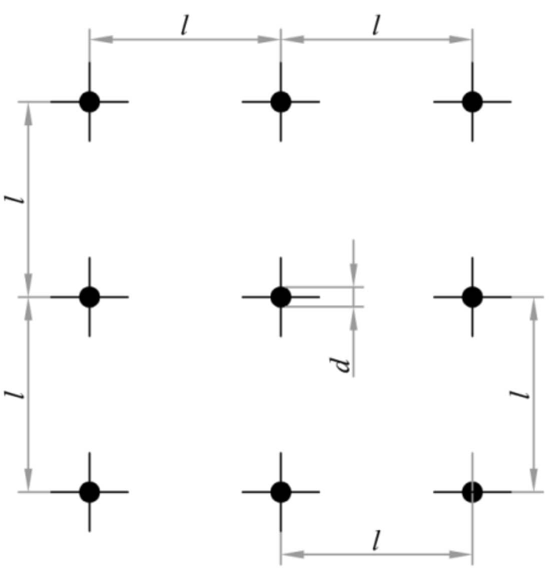
$$N = n_1 \cdot n_2 = \frac{b \cdot V_S \cdot t}{l^2}$$

So the sediment discharge per unit width is given by:

$$q_S = \frac{Q_S}{b} = \frac{N \cdot W}{b \cdot t} = \frac{V_S \cdot W}{l^2} = V_S \cdot d \cdot \frac{W}{l^2 \cdot d} = V_S \cdot d \cdot m \quad (1)$$

d sediment particle diameter

$m = \alpha d^2/l^2$ dynamic coefficient of continuity (ratio of the particle volume to the volume of the layer where the particles move) (α depending on the particle shape)



Sketch of the particle distribution

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Background (3)

It is also assumed that *m* is a function of the main-flow section-averaged velocity *V*:

$$m = \frac{\alpha \cdot d^2}{l^2} = f(V)$$

The sediment particle velocity V_S is usually expressed as a function of the water velocity *V* and the critical velocity V_0 :

$$V_s = \varphi (V - V_0)$$

where φ is a constant coefficient.

The main assumption of Levi is that *m* can be cast as the product of $\frac{V}{\sqrt{gd}}$ by a function of $k' = h/d$ (or, equivalently, of $k = d/h$), being *h* the flow depth:

$$m = f\left(\frac{h}{d}\right) \cdot \left(\frac{V}{\sqrt{gd}}\right)^3$$

where the function $f\left(\frac{h}{d}\right)$ has a typical exponential form $\beta \left(\frac{d}{h}\right)^n$, with *n* = 0.25.

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Background (4)

The sediment discharge per unit width in equation (1) can then be rewritten as:

$$q_s = \varphi\beta\left(\frac{V}{\sqrt{gd}}\right)^3 d\left(V - V_0\right)\cdot\left(\frac{d}{h}\right)^n$$

The ratio μ_C between sediment and water volumetric discharges per unit width is:

$$\mu_C = \frac{q_s}{q} = C\left(\frac{V}{\sqrt{gd}}\right)^3\left(1 - \frac{V_0}{V}\right)\left(\frac{d}{h}\right)^{n+1}$$

with $C = \beta\varphi$.

Taking into account that the critical velocity V_0 can be written (Studenitcnikov, 1964) as:

$$V_0 = A\sqrt{g\Delta}\left(hd\right)^{0.25}$$

where $\Delta = \frac{\gamma_s - \gamma}{\gamma}$ is the relative density of the submerged grains, then:

$$\mu_C = C\left(\frac{V}{\sqrt{gd}}\right)^3\left(1 - \frac{A\sqrt{gd\Delta}}{V}\left(\frac{h}{d}\right)^{0.25}\right)\left(\frac{d}{h}\right)^{1.25}$$

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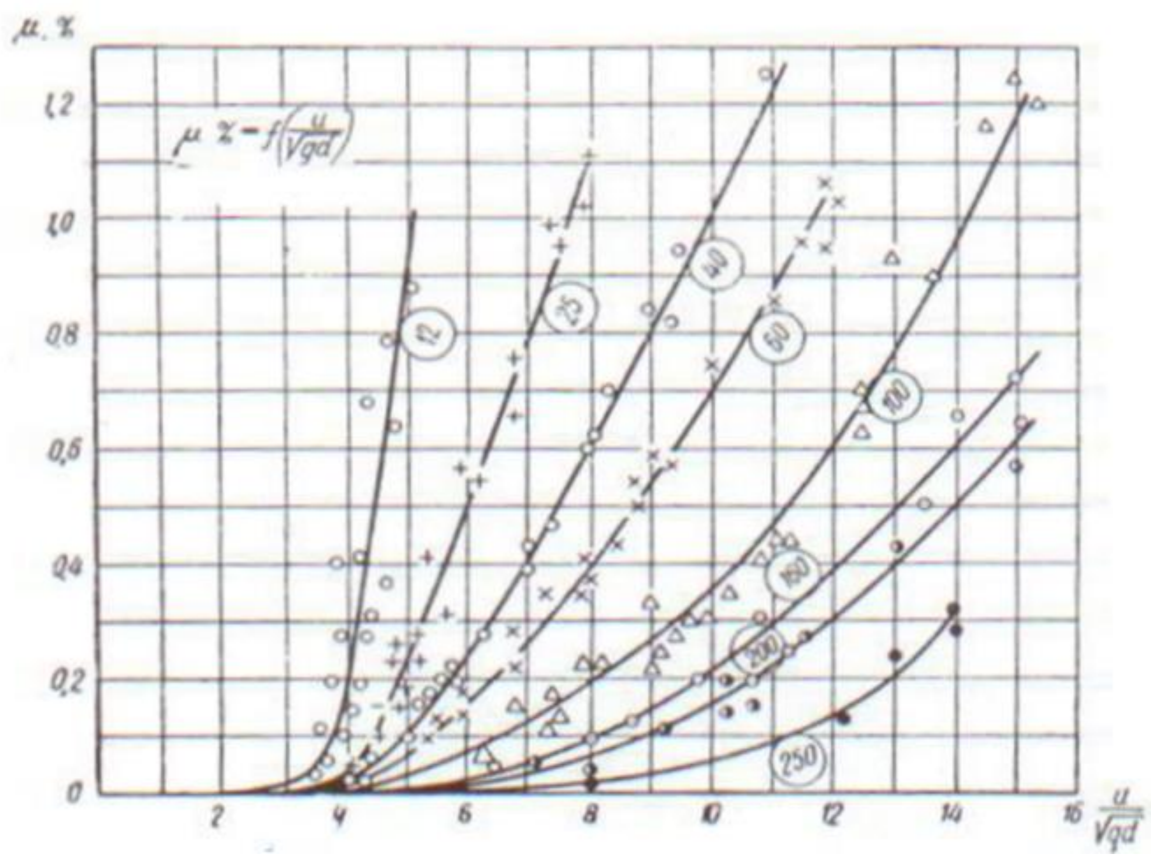
Background (5)



Experimental results in term of curves $\mu_c \% = f\left(\frac{V}{\sqrt{gd}}\right)$
for different ratios $k' = h/d$
as given by Levi (1948)

The Levi's formula

Results of fitting the curves to the data provide the following expression for the solid discharge in volumetric units:

$$q_s = 2.03 \cdot 10^{-7} \left(1 - \frac{V_0}{V}\right) \frac{V^4}{g\sqrt{g\sqrt{dh}}} \quad [m^2/s]$$

This equation is valid for ratios $k' < 500$, although its validity can be extended up to values of $k' < 5000$.

Dimensionless formulation (1)

The solid discharge can be also expressed in a dimensionless form.

Besides the relative particle *k* (or equivalent *k'*) and the relative density Δ, some additional quantities can be defined:

- the Shields's mobility parameter $\theta = \dfrac{\tau_0}{(\gamma_s - \gamma)d}$
- the shear velocity of the flow $u^* = \sqrt{\theta g d \Delta}$
- $\Phi = \dfrac{q_s}{\sqrt{g d \Delta} \cdot d}$ from which $q_s = \Phi \sqrt{g d \Delta} \cdot d$

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Dimensionless formulation (2)

The velocity V can be expressed as:

$$V = C_h \cdot u^*$$

where the dimensionless Chezy coefficient $C_h = K_{Ch} / \sqrt{g}$ (K_{Ch} dimensional Chezy coefficient) can be evaluated as a function of k', through one of the formulas available in the technical literature, e.g. the one by Bray and Davar (1987):

$$C_h = 2.48 \log \frac{h}{d} + 3.1$$

The dimensionless variable Φ, computed through the previously found expression for q_s, becomes a function of k' and θ (or, equivalently, of k) by replacing this formula and the definition of u^* and k' into the expression of V.
It is then possible to plot the values of Φ against θ for fixed values of k', also comparing against the reference values of the function Φ_r evaluated according to the formula of Meyer-Peter and Müller (1948):

$$\Phi_r = 8(\theta - 0.047)^{3/2}$$

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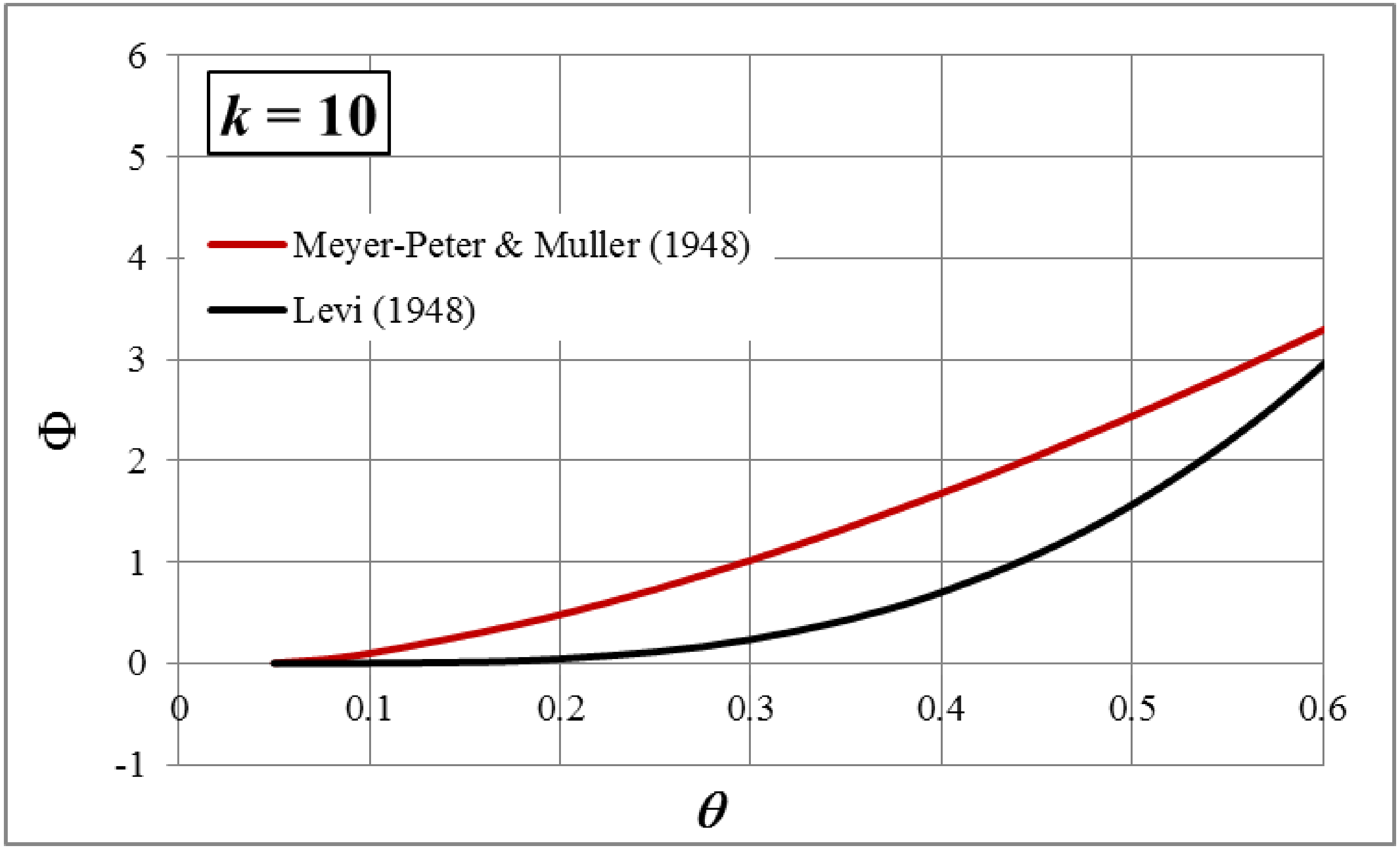
LEVI'S FORMULA FOR SEDIMENT DISCHARGE



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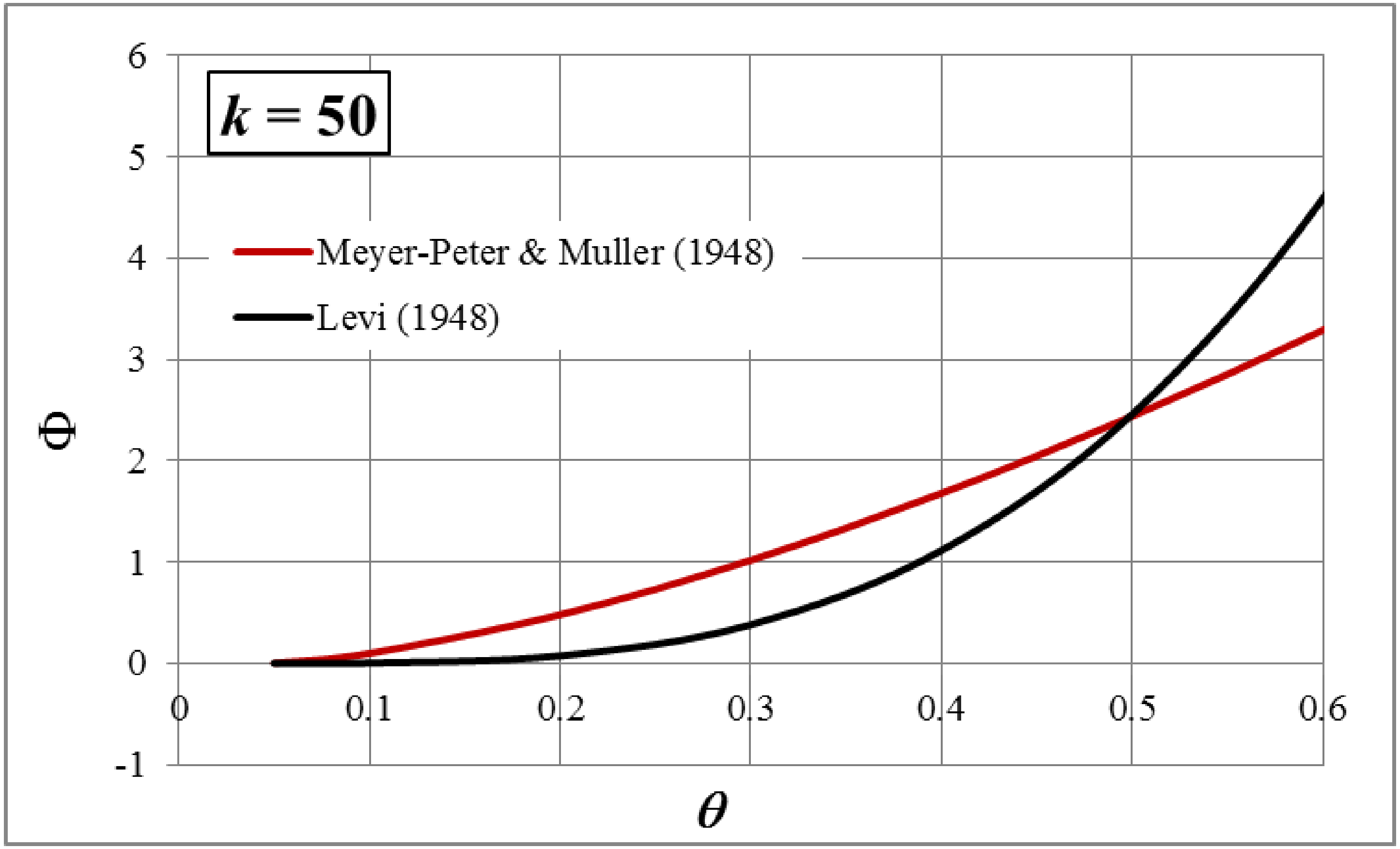
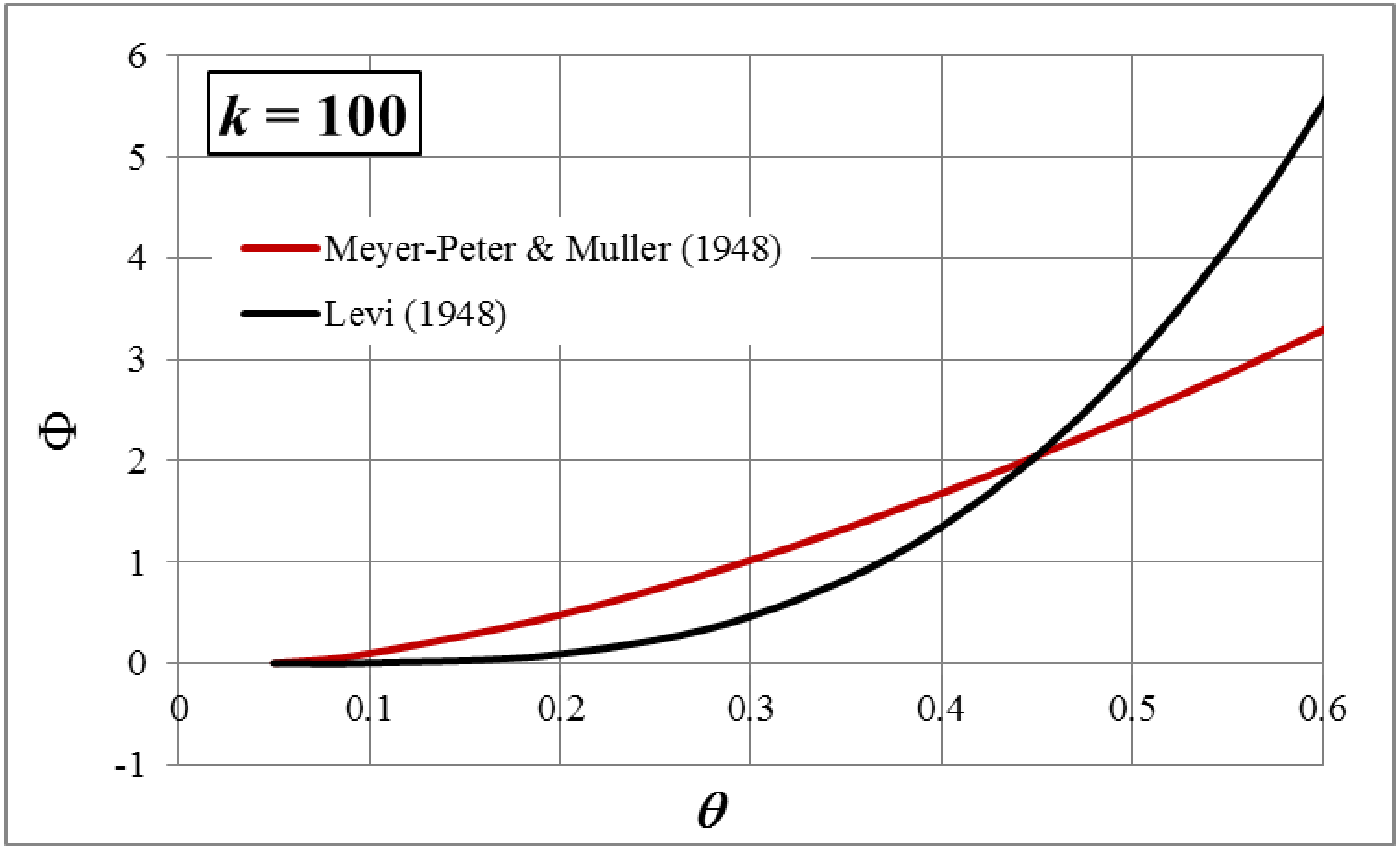


LEVI'S FORMULA FOR SEDIMENT DISCHARGE



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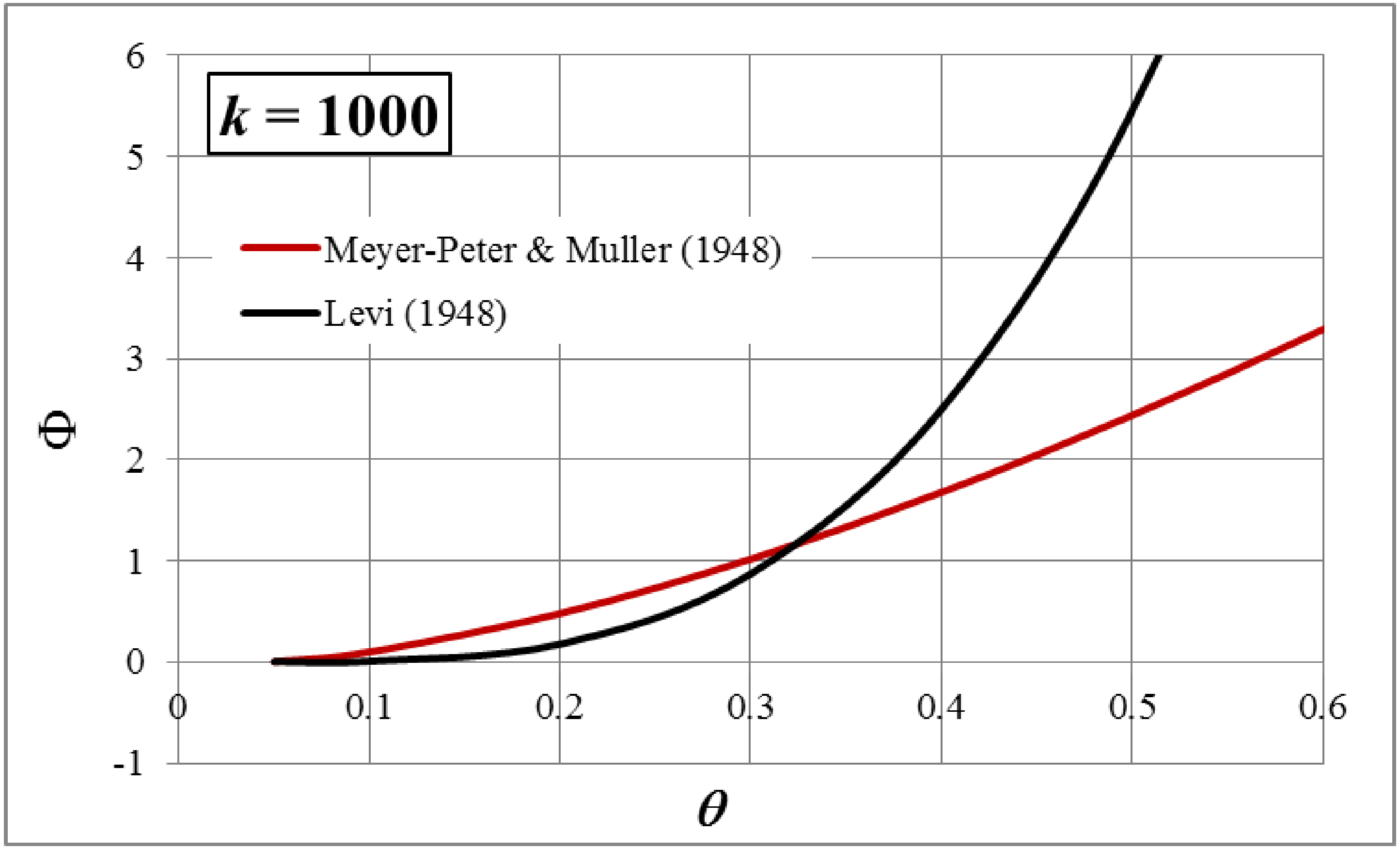
LEVI'S FORMULA FOR SEDIMENT DISCHARGE



Remarks

The values of θ considered in the plots do not exceed 0.6, value approximately corresponding to the lower limit of the upper flow regime; higher values would correspond to the formation of antidunes (Engelund, 1965) and Levi's assumption of flat bed would not be acceptable anymore.

Plots show that, in this range of Shields numbers, the application of the Levi's formula (1969) leads to results which do not differ from the ones obtained by the classic formula of Meyer-Peter and Müller (1948) more than the ones obtained through other formulae typically adopted in the Western hydraulics school.

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STUDENITCNIKOV'S FORMULA FOR CRITICAL VELOCITY

Studenitcnikov spent his work life as a researcher in VNII Vodgeo (Moskow, Russia), a leading research, design and technology development institution on water engineering and environmental protection.

One of his most interesting contribution in the field of Hydraulics was his campaign of experiments aimed at finding a *critical velocity for incipient sediment motion* and performed in the hydraulic laboratory of VNII Vogeo in the years 1955-1963.

Laboratory data were further extended by processing field data of Russian rivers. The result, compounded in a simple formula, was widely used in Russia for the design and construction of the hydrotechnical structures.

One of the approaches to sediment transport starts, in fact, by defining a threshold for the incipient mobility of grains. The well known and widely used criterion of Shields is almost standard in the western literature. In the former Soviet Union a similar status was achieved by the approach of B. Studenitcnikov.

Background (1)

A current flowing on an erodible bottom tends to transport the bed material downstream.

A submerged grain on the bed is subjected at the same time to a weight force and a hydrodynamic force. Under some critical hydraulic condition, the latter is so small that particles are not able to move. However, a slight increase in the flow velocity above this critical condition may initiate appreciable motion of some of the particles on the bed.

This hydraulic critical condition is named the *condition of initiation of motion* (or *incipient motion*) and is computed in terms of either mean flow velocity or critical bed shear stress.

The incipient motion state for bed material, in steady uniform-flow conditions, is determined by Studenitcnikov through the critical cross-section averaged velocity V_0, and is futhermore assumed to depend on the following parameters:

- γ specific weight of water
- γ_s specific weight of solid particles
- d particle mean diameter
- h flow depth

As a reference, flow in a rigid rectangular channel with a large cross-section (width to height ratio $B/h \geq 2.5\text{-}3.0$) is considered, with normal turbulence and velocity distribution along the depth.

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Background (2)

Assuming a power law for the velocity, the lifting force acting on the particle may be written as:

$$F_z = K_1 \cdot \gamma \frac{\alpha V_0^2}{2g}\left(\frac{d}{h}\right)^{2n} \frac{\pi d^2}{4}$$

where: K_1 coefficient of proportionality

$\alpha \cong 1$ Coriolis coefficient for the velocity head

g gravity acceleration

n exponent for the velocity distribution law

The submerged weight of the single solid particle in water is equal to:

$$G = K_2(\gamma_s - \gamma)\frac{\pi d^3}{6}$$

where K_2 is a shape coefficient.

The limit condition for the stability of the particle is, then, obtained if:

$$K_1 \cdot \gamma \frac{\alpha V_0^2}{2g}\left(\frac{d}{h}\right)^{2n} \frac{\pi d^2}{4} = K_2 \cdot (\gamma_s - \gamma)\frac{\pi d^3}{6}$$

which means when the velocity is critical, and then equal to:

$$V_0 = A\sqrt{g\Delta} \cdot h^n d^{0.5-n} \qquad (2)$$

with: $A = 2\sqrt{\frac{K_2}{3\alpha K_1}}$ $\Delta = \frac{\gamma_s - \gamma}{\gamma}$

Background (3)

The best interpolation of the experimental data is achieved by n = 0.25.

This value for n may also be justified, according to Studenitcnikov, by considering that equation (2) shows that the critical velocity V_0 depends on d and h. Introducing the relative size of the particles $k = d/h$, dividing by d^n, the expression of V_0 becomes:

$$V_0 = A\sqrt{g\Delta}\cdot k^{0.5-2n}\left(h^{0.5-n}d^n\right)$$

In order to avoid the explicit dependency on k, the exponent of k has to be zero ($k^{0.5-2n} = 1$).

Hence, a condition for n is found: 0.5 – 2n = 0, by which n = 0.25, and thus:

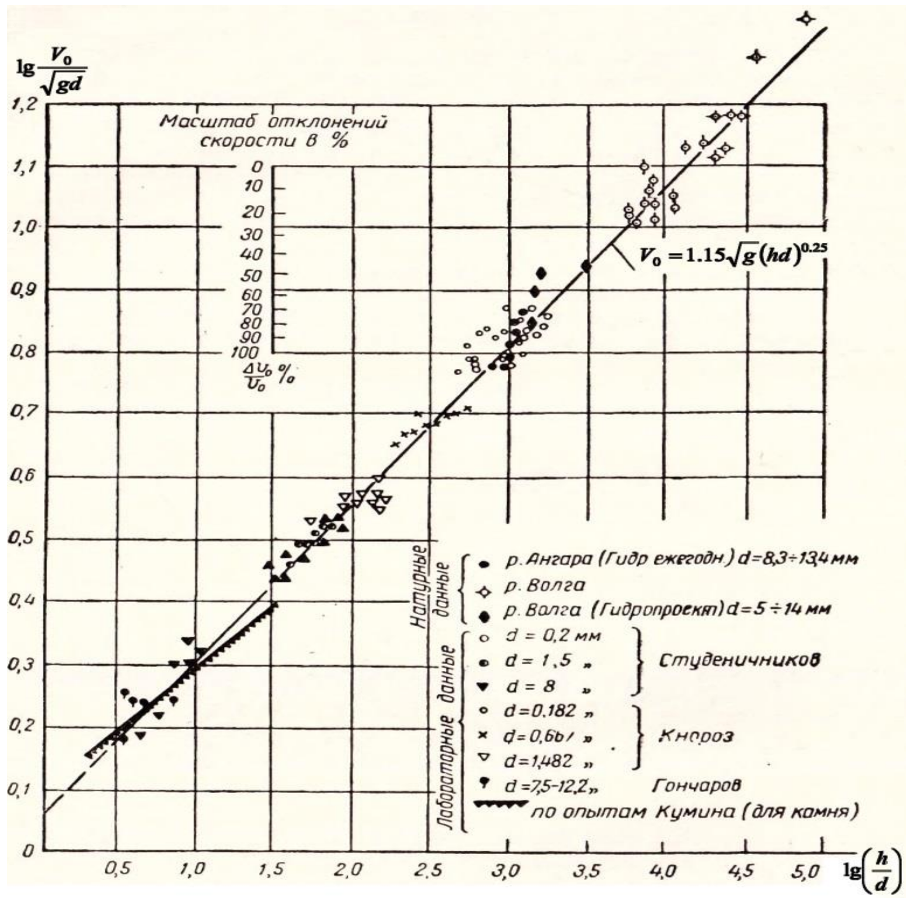
$$A\sqrt{g\Delta} = \frac{V_0}{h^{0.25}d^{0.25}} = f(k) = const$$

The amount $B = A\sqrt{\Delta} = \frac{V_0}{\sqrt{g}\cdot(hd)^{0.25}}$ is, therefore, a dimensionless variable.

Processing experimental data in a wide range of relative flow depth $k' = h/d$, values of B = 1.15 and, therefore, of A = 0.9, are obtained for γ_s = 2.65 and γ = 1 (Δ = 1.65). The critical velocity V_0, then, can be finally written as:

$$V_0 = 0.9\sqrt{g\Delta}(hd)^{0.25} = 1.15\sqrt{g}(hd)^{0.25}$$ or: $$V_0 = \frac{0.9}{k^{0.25}}\sqrt{gd\Delta}$$

Background (4)



Experimental results for Δ = 1.65 in terms of $V_0 / \sqrt{gd}$
as a function of $k' = h/d$
as given by Studenitcnikov

Dimensionless form (1)

In the Western literature the threshold condition for sediment motion is usually derived according to the Shields' theory (Shields, 1936).

In order to compare the results obtained by Studenitcnikov with the Western equivalent ones, a dimensional analysis is useful.

Besides the relative particle *k* (or equivalent *k*') and the relative density Δ, the Shields's mobility parameter is introduced:

$$\theta = \frac{\tau_0}{(\gamma_s - \gamma)d} = \frac{(u^*)^2}{gd\Delta} \quad \Rightarrow \quad u^* = \sqrt{gd\Delta\theta}$$

The flow average velocity *V* can be evaluated, through one of the different literature formulae for uniform-flow conditions, for example the Chezy formula, as:

$$V = K_{Ch} \cdot \sqrt{RJ} \qquad \text{or} \qquad V = C_h \cdot u^*$$

$$\Rightarrow \quad \theta = \frac{V_0^2}{C_h^2 gd\Delta}$$

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Dimensionless form (2)

According to the Shields' theory, the motion of the solid particles will start when the tangential stress will reach a critical value, i.e. when the dimensionless parameter θ will reach a critical value θ_c.

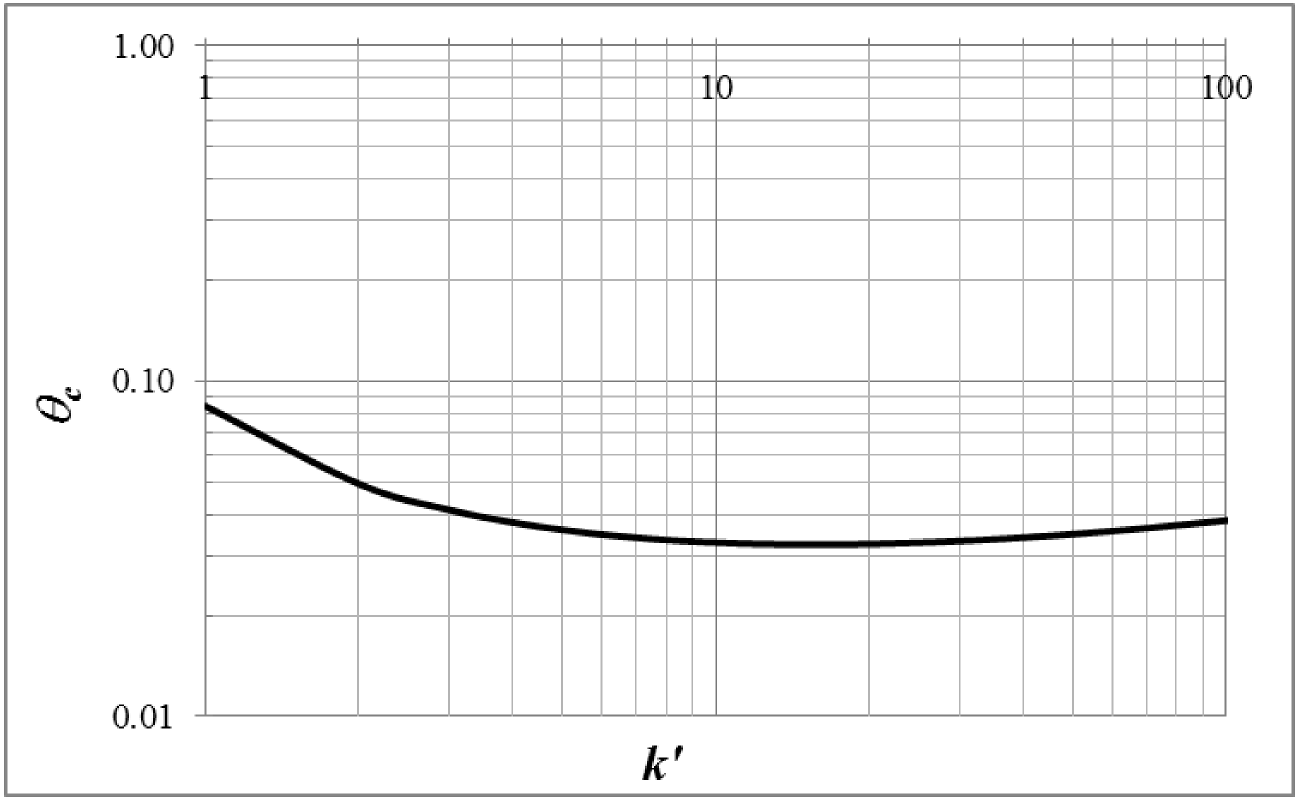
This critical value $\theta_c = \dfrac{V_0^2}{C_h^2 g d \Delta}$, substituting the expression by Studenicnikov for the critical velocity V_0, becomes:

$$\theta_c = \frac{0.81}{\sqrt{k} \cdot C_h^2}$$

As a consequence, when the Chezy coefficient C_h is evaluated through one of the formulae available in the technical literature, as a function of the relative water depth *k'*, the critical Shields' number becomes a univocal function of *k*, or, equivalently, of *k'*:

$$\theta_c = \frac{0.81 \cdot \sqrt{k'}}{C_h^2}$$

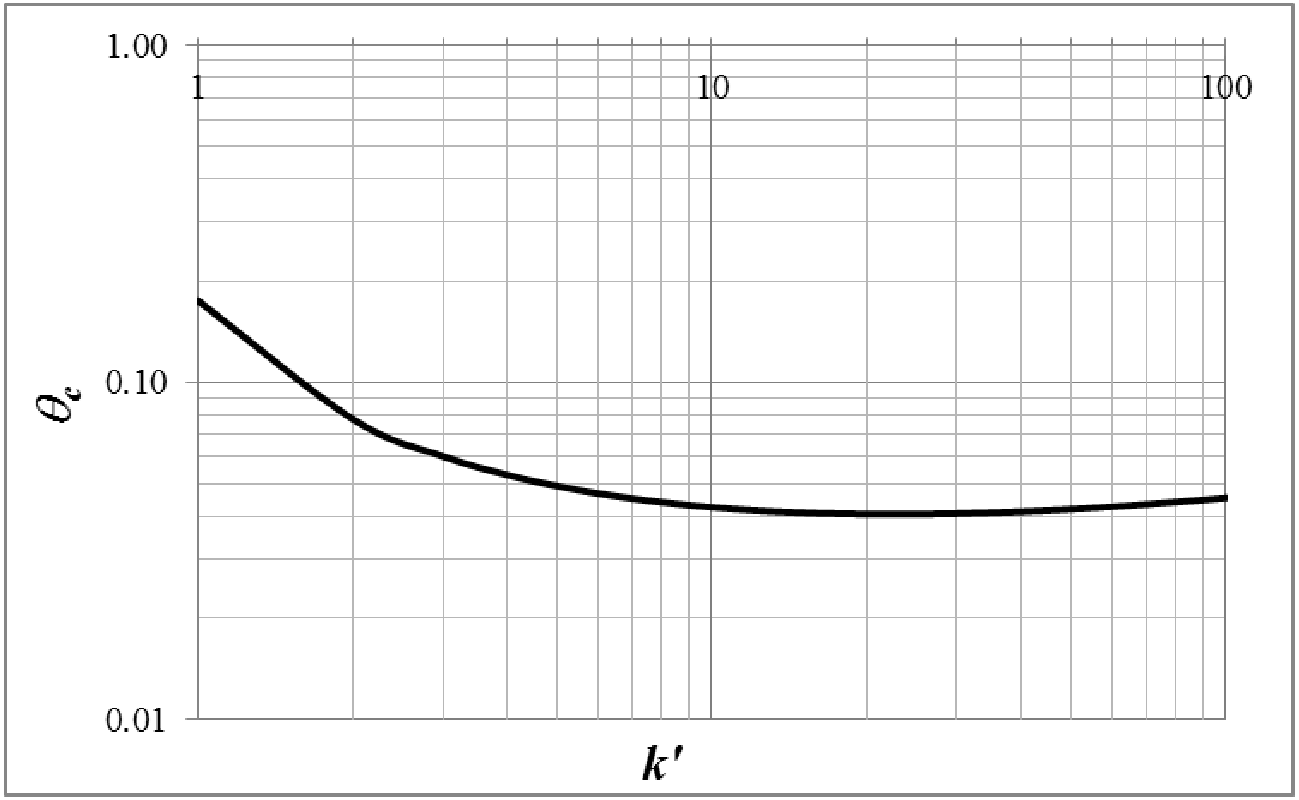
Dimensionless form (3)



Values of θ_c as a function of k' calculated assuming C_h by the formula of Bray and Davar (1987):

$$C_h = 2.48 \ln \frac{h}{d} + 3.1$$

Dimensionless form (4)



Values of θ_c as a function of k' calculated assuming C_h by the formula of Griffits (1981):

$$C_h = 2.43 \ln \frac{h}{d} + 2.15$$

Remarks

Figures show that the values of θ_c computed by Studenitchikov's formula are in the same range of those predicted by the better-known (in the West) Shields' criterion

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Conclusions

While there are quantitative differences between the sediment discharges predicted by Levi formula and the other, these differences are about in the same range as those that the comparison between any two different formulas will show. Levi formula, by its use of the grain size to depth ratio and the depth averaged local velocity, may result easier to use in geo-morphological models. Its dependency on the fourth power of velocity matches some other formulas better known in the West, like the van Rijn one. The additional dependency by the *h/d* parameter may increase its ability to interpret a wider range of experiments.

If the critical "incipient motion" velocity in it is computed according to Studenicnikov no further dependency is needed. It should be noted, anyway, that Studenitcnikov critical conditions do NOT exhibit a constant θ_c value for large *h/d*.

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Thanks

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